A CALCULATION PROCEDURE FOR TWO PHASE FLOW DISTRIBUTION IN MANIFOLDS WITH AND WITHOUT HEAT TRANSFER

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Abstract-A finite-difference procedure has been developed forca1culating two phase flows with heat transfer in parallel,reverse and mixed flowmanifolds. The flowsare assumed to be homogeneous and one dimensional. An iterative numerical scheme is used to solve the differential equations for longitudinal momentum and continuity, while in the lateral direction an integral equation for momentum is solved. Numerical predictions compare well with the available experimental results for a combining flow manifold. The effect of total flow rate, heat input on the lateral flow distribution and pressure drop characteristics have been demonstrated.

NOMENCLATURE

Greek symbols

ρ	density
μ	viscosity

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Superscripts

1. INTRODUCTION

A MANIFOLD can be described as a flow channel (commonly known as header) having a number of discrete openings in the side walls (known as laterals) through which the fluid enters or leaves the header. As illustrated in Fig. 1, simple dividing and combining flow manifolds are the two basic types of manifold. Parallel, reverse and mixed flowarrangements are combinations of the basic dividing and combining flow manifolds. Parallel, reverse and mixed flow arrangements are commonly used in steam generators for heating water to steam. The water is distributed in the lateral branches through the dividing flow header and is heated while flowing through the branch pipes. The combining flow header collects the water and steam mixture.

A designer often needs to know the exit quality of steam and the pressure drop, as well as the extent of non-uniformity in flows through lateral branches for a given heat input.

The flow field in the header of manifold systems can be regarded as l-dim, for many practical purposes. However, due to the elliptic nature of the flow in the header, the Bernoulli equation cannot be applied. On

the other hand, it is necessary to solve simultaneously (I) the longitudinal momentum equation and (2) the continuity equation in the header, and (3) the discharge equation in the lateral branches to obtain the static pressure and the two components of velocity. Early authors $\lceil 1-3 \rceil$ combined the three equations to obtain one ordinary differential equation called the flow distribution equation. With the appropriate boundary condition,Bajura [1] obtained an analytical solution of the flow distribution equation for frictionless flows . For viscous flows an iterative solution has been obtained [2, 3] by matching the given boundary conditions. However, in such analytical models $[1-3]$, the headers were assumed to be uniformly porous instead of having discrete lateral branches. Since these formulations do not allow for any property variation, an extension ofthe calculation scheme to the prediction of two phase flows cannot be developed. Recently one of the present authors [4] developed an iterative numerical procedure for calculating flow distribution in a dividing and combining flow manifold; this has since been extended [5] by the present authors to cover the calculation of flows in parallel and reverse flow manifolds. The iterative numerical scheme originated in the SIMPLE (Semi Implicit Pressure Linked Equation) algorithm of Patankar and Spalding [6]. Unlike earlier analytical models, the proposed numerical scheme [4] can allow for property variation in the flow domain and for a more realistic representation oflateral branches in the mathematical model.

The present paper is concerned with the further de velopment of the numerical scheme [4, 5] for predicting homogeneous two phase flows with heat transfer. The method is first tested by predicting with reasonable accuracy the experimental data [7] in a combining flow manifold for an air-water mixture. Then heat transfer and flow calculations are performed for parallel, reverse and mixed flow manifolds with different input variables.

2. PREDICfION PROCEDURE

Gocerninq equations

The equation of motion governing the flows in the systems shown in Fig. 1 can be written as follows:

longitudinal momentum equation for headers

$$
C_{\text{T}}\rho u \frac{\mathrm{d}u}{\mathrm{d}x} = -\frac{\mathrm{d}p}{\mathrm{d}x} - \frac{f}{8} \frac{\rho u^2 S}{a_{\text{H}}} \tag{1}
$$

continuity equation for headers

$$
\frac{\mathrm{d}}{\mathrm{d}x}(\rho u a_{\mathrm{H}}) = q/l \tag{2}
$$

where

$$
q = KGa_{\mathsf{L}} \tag{3}
$$

for dividing flow header $K = -1$, and for combining flow header $K = 1$.

A new coefficient C_T , called the turning loss

coefficient, is introduced in the convective term of equation (1) to account for the change in pressure due to turning ofthe fluid stream.The physical significance of C_T has been explained [4]. Kubo and Ueda [7] determined the values of C_T by performing experiments in combining flow manifold with an air-water mixture. The last term on the RHS of equation (I) represents fluid friction.

The RHS of equation (2) can be regarded as the source term of the continuity equation. In a dividing flow manifold the source term is a negative quantity as the control volume is losing fluid through lateral branches whereas it becomes positive in a combining flow manifold due to the entry of more fluid.

The liquid in the lateral tubes is heated uniformly over its length by a uniform heat flux. Itis assumed that a length of the tube, L_{SP} , is required to preheat the liquid to saturation temperature. The rest of the tube contains a mixture of vapour and liquid. This two phase flow has been analysed by homogeneous model flow assumptions. Under the homogeneous flow model assumption for two phase flow, the equation connecting the pressure drop and the flow rate in the lateral pipes is given by [8]

$$
\Delta P = G^2 \left[\frac{2f_{\rm SP}L_{\rm SP}v_{\rm f}}{D} + \frac{2f_{\rm TP}L_{\rm TP}v_{\rm f}}{D} + (v_{\rm f} - v_{\rm fi}) + v_{\rm fg}x_0 \right] + \frac{gL_{\rm SP}}{v_{\rm f}} + \frac{gL_{\rm TP}}{v_{\rm fg}x_0} \ln \left(1 + x_0 \frac{v_{\rm fg}}{v_{\rm f}} \right). \tag{4}
$$

The first and second terms of equation (4) denote the frictional pressure drop for single phase and two phase conditions, respectively, where the friction coefficients f_{SP} and f_{TP} are given by

$$
f_{\rm SP} = 0.079 \left(\frac{GD}{\mu_f}\right)^{-0.25},\tag{5}
$$

$$
f_{\rm TP} = 0.079 \left(\frac{GD}{\mu_{\rm TP}}\right)^{-0.25},\tag{6}
$$

$$
\mu_{\rm TP} = x_0 \mu_{\rm g} + (1 - x_0) \mu_{\rm f}.\tag{7}
$$

The quantities L_{SP} and L_{TP} are calculated according to the following formulas. The enthalpy rise across the tube is

$$
\Delta i = \frac{\text{heat input}}{\text{water flow rate}}.
$$
 (8)

The length of the tube with single phase flow, L_{SP} , is given by

enthalpy of saturated liquid L_{SP} -enthalpy of inlet water

$$
\frac{1}{L} = \frac{1}{\Delta i}, \tag{9}
$$

$$
L_{\rm TP} = L - L_{\rm SP}.\tag{10}
$$

The acceleration of the water due to a change in specific volume produces a small pressure drop. This drop has been accounted for in the third term of equation (4). The fourth term of equation (4) represents acceleration pressure drop for two phase flow. The fifth and sixth terms of the same equation denote the single phase and two phase gravitational head, respectively.

Boundary conditions

The velocities at the two ends $(x = 0$ and $x = l$, Fig. 1) for different configurations of the headers are given as follows:

Dividing flow header at

$$
x = 0, \quad u = u_{\text{inlet}} = \frac{Q}{\rho a_{\text{H}}} \tag{11}
$$

and at

$$
x = l, \quad u = 0 \tag{12}
$$

combining flow header

(i) for parallel flow at

$$
x = 0, \quad u = 0 \tag{13}
$$

and at

$$
x = l, \quad u = u_{\text{exit}} = Q/\rho a_{\text{H}}
$$
\n
$$
\tag{14}
$$

(ii) for reverse flow at

$$
x = 0, \quad u = u_{\text{exit}} = Q/\rho a_{\text{H}} \tag{15}
$$

and at

$$
x = l, \quad u = 0 \tag{16}
$$

(iii) for mixed flow at

$$
x = 0, \quad u = 0 \tag{17}
$$

and at

$$
x = l, \quad u = 0. \tag{18}
$$

Grid

In the present numerical scheme, the 'staggered grid' arrangement has been employed. Such grid arrange- where ments have been applied earlier by Majumdar [4] and Datta and Majumdar [5] in the analysis ofsingle phase manifold problems.The arrangement isshown in Fig. I. The main features of this system are that the velocities are located between two pressure points and other scaler variables, if any, are located at pressure points. This has the following two advantages over the conventional system, where all scalar and vector properties of the fluid are located at the same place:

(a) this is conceptually simple, i.e. the pressure difference driving the velocity; and

(b) velocities are available at points where they are needed for the mass balance around a control volume.

Finite-difference equation

Integrating the momentum equation (1) over a control volume as shown in Fig. 2(a), the finite difference equation can be written as

$$
D_i^u u_i = A_i^u u_{i+1} + B_i^u u_{i-1} + a_{11}(p_i - p_{i+1}) + C_i^u \quad (19)
$$

FIG. 1. Schematic diagram for (a) parallel flow manifold, (b) reverse flow manifold, and (c) mixed flow manifold.

$$
A_i^u = -\frac{C_{\rm T}}{8} \left[(\rho u a_{\rm H})_i + (\rho u a_{\rm H})_{i+1} \right], \tag{20}
$$

$$
B_i^u = \frac{C_{\rm T}}{8} \left[(\rho u a_{\rm H})_i + (\rho u a_{\rm H})_{i-1} \right],\tag{21}
$$

$$
C_i^u = -\frac{f}{8} \rho u_i^2 S_i \Delta x_i, \qquad (22)
$$

$$
D_i^u = -(A_i^u + B_i^u), \t\t(23)
$$

u and *P*are the velocity and pressure respectively in the concerned header. According to homogeneous flow theory, average density for two phase flows are determined.The control volume for continuity appears in Fig. 2(b). Mass conservation equation in finite difference form can be written as

$$
(\rho a_{\rm H} u)_i - q_i - (\rho a_{\rm H} u)_{i-1} = 0. \tag{24}
$$

In the present numerical scheme, a straightforward use of equation (24) will not be made. Instead a pressure

and

FIG. 2. Control volume for (a) momentum and (b) continuity cell.

correction equation will be derived from this equation as suggested by Patankar and Spalding [6] in their SIMPLE (Semi Implicit Pressure Linked Equation) algorithm. According to SIMPLE, the purpose of obtaining the pressure correction equations are two fold. First, pressures are to be corrected. Secondly, velocities are corrected through a linearized momentum equation so as to satisfy the continuity. Later Spalding [9] proposed a modification (SNIP) to SIMPLE where he suggested using a pressure correction to correct velocities only. The pressures are determined by a new integration of the momentum equation while integrating the pressures, corrected velocities should be used. The present scheme employs the SNIP as outlined below.

Solution procedure

The present numerical scheme consists of the following sequence of calculations:

(1) Initially the prescribed total rate, Q , is distributed uniformly through each of the lateral tubes. To start the iteration procedure the pressures (p^d and p^c) are initially guessed.

(2) The dividing flow header is considered first and the following iterative procedure has been employed:

(i) The finite difference equation for momentum [equation (19)] is solved by Tri-Diagonal Matrix Algorithm (TDMA) to generate an approximate velocity field u^* . During the iterative cycle the pressures obtained in the previous cycle are used for calculating u^* .

(ii) The u^* values are in general not compatible with continuity because an incorrect pressure field has been used. Therefore the following correction equations for pressures and velocities are derived from continuity (derivation appears in Appendix).

Pressure correction equation.

$$
D_i^{\rm p} p_i' = A_i^{\rm p} p_{i+1}' + B_i^{\rm p} p_{i-1}' + C_i^{\rm p} \tag{25}
$$

where

$$
A_i^{\rm p} = \left(\frac{\rho a_{\rm H}^2}{D^u}\right)_i,\tag{26}
$$

$$
B_i^{\mathbf{p}} = \left(\frac{\rho a_{\mathbf{H}}^2}{D^u}\right)_{i=1},\tag{27}
$$

$$
C_i^{\rm p} = (\rho a_{\rm H} u^*)_{i-1} - (\rho a_{\rm H} u^*)_{i} - q_i, \tag{28}
$$

$$
D_i^{\rm p} = A_i^{\rm p} + B_i^{\rm p}.\tag{29}
$$

Velocity correction equation.

$$
u'_{i} = \left(\frac{a_{\rm H}}{D^{u}}\right)_{i} (p'_{i} - p'_{i+1})
$$
 (30)

$$
u_i = u_i^* + u_i'.
$$
 (31)

(iii) Equation (25) is solved by TDMA for the pressure corrections. Subsequently velocities are corrected by equations (30) and (31) to bring them into conformity with the continuity equation. With the corrected velocities, the pressures are now calculated using equation (19).

(iv) Steps (i}-(iii) are repeated until the momentum and continuity errors are reduced to a pre-assigned small value. Fractional errors of momentum, RS_{mom} , and continuity, RS_{con} , are defined as

$$
RS_{\text{mom}} = \frac{\sum_{i=1}^{n} [D_i^u u_i - A_i^u u_{i+1} - B_i^u u_{i-1} - a_{\text{H}}(p_i - p_{i+1}) - C_i^u]}{(\rho u^2)_{\text{inlet}} a_{\text{H}}}
$$
\n(32)

and

$$
RS_{con} = \frac{\sum_{i=1}^{n} |a_{H}\rho u_{i-1} - \rho a_{H} u_{i} - q_{i}|}{(\rho u)_{\text{inlet}} a_{H}}.
$$
 (33)

(3) With the same lateral flow distribution *qi,* the combining flow header is now considered. By repeating the steps $2(i)$ to $2(iii)$ for combining the flow header, the longitudinal velocities, u^c, and pressures, p^c, ard calculated.

(4) The pressure drop, Δp , in the lateral pipe is now determined from the difference of p^d and p^e . Using equation (4), a new lateral flow distribution q_t^* is now calculated.

(5) It is likely that the predicted lateral flow distribution, q_t^* , will not conform to the prescribed flow rate Q as the pressures (p^d and p^c) which are used for the calculation of *qt,* are based on some guessed lateral flow distribution. The error, ΔQ , can be written as

$$
\Delta Q = Q - \sum_{i=1}^{n} q_i^*.
$$
 (34)

(6) The lateral flows are to be corrected to eliminate ΔQ ; a uniform correction is applied to all q_i^* 's.

$$
q_i = q_i^* + \frac{\Delta Q}{n}.\tag{35}
$$

Pressures in the dividing flow header, p^d , are once again calculated employing equation (4).

FIG. 3. Comparison of the predicted lateral flow distribution with the measurements of Kubo and Ueda [7].

(7) Steps (2)–(6) are repeated until $\Delta Q/Q$ reduces to a pre-assigned negligible value.

It may be mentioned here that the same calculation procedure with little modifications can be applied for a simple dividing flow manifold or a simple combining flowmanifold.The simple manifolds consist of a single header and one of the two pressures p^d and p^c is calied p_{ref} . The pressure drop Δp will be defined as $\Delta p =$ $(p^d - p_{ref})$ or $(p_{ref} - p^c)$. The details are given in ref. [4].

3. RESULTS AND DISCUSSION

Computational details

In all computations described below, the convergence criterion (RS_{mom} , RS_{con} and $\Delta Q/Q$) were set at 0.005.The numberofiterations necessary to achieve the above criterion were mainly dependent on the type of manifold and dimensionless parameters. The prediction of a typical reverse flow system (situation as mentioned in Fig. 4 with $Q = 0.684$ kg s⁻¹) requires 5 iterations (steps 2-7).

Experimental data ofKubo and Ueda

Kubo and Veda [7] reported some experimental data on the flow rate distribution of an air-water mixture in a combining flow manifold. Figure 3 provides a comparison of the lateral flow distribution between the present numerical scheme and measurements of Kubo and Veda [7]. It is observed that the agreement is generally satisfactory. The values of turning loss coefficient, C_T and the resistance coefficient for discharge through branch pipes, C_d in the present computational scheme have been taken from the experiments of Kubo and Veda . Kubo and Veda also presented an iterative procedure for calculating the lateral flow distribution.They have mentioned in their report that their calculated values for fractional flow rates fall below the experimental results at the down stream and near the exit of the header. However, the present method predicts the measured values at these points quite reasonably.

!If*anifolds for steam generations*

Three different types of manifolds, namely reverse, parallel and mixed flow manifolds as shown in Fig. 1, have been considered to test the performance of the present numerical scheme. No published literature concerning experimental investigations for such manifolds was available for comparison. A problem as

Table I. Input data for a steam generating system

Inside diameter of the headers	$25 \,\mathrm{mm}$
No. of lateral tubes	6
Inside diameter of the tubes	10.2 mm
Height of lateral tubes	3.66 _m
Inlet temperature of water	204° C
Pressure of water-steam mixture at the	
outlet header	68.9 bar
Heat absorbed in the panel consisting	
of lateral tubes	50, 100, 150, 200 kW
Total flow rate	0.684, 1.296, 1.944 kg s ⁻¹

FIG.4. Fractional flow and dryness fraction in the branch pipe for the reverse flow manifold.

FIG.5. Fractional flow and dryness fraction in the branch pipe for the parallel flow manifold.

FIG.6. Total pressure drop vs heat load for different manifold systems.

stated in Table 1 has been solved for three different types of manifolds. The dividing flow header is filled with unsaturated hot water. As the fluid passes through the lateral tubes it receives heat from the furnace and steam formation occurs if the heating is sufficient.

Figure 4 shows the predicted fractional flow distributions and steam quality for 100 kW heat load and for two different flow rates. For a given heat load, non-uniformity in lateral flow distribution increases with increase offIow rate. Drynessfraction increases, as expected, with the reduction in flow rate .The predicted fractional flow distributions and dryness fraction for a parallel flow manifold are shown in Fig. 5. In case of higher flow rate there is no steam formation in lateral numbers 5 and 6. This seems to be the reason for the steep change in slope in the predicted lateral flow distribution. The predicted relationship between the pressure drop and heat load is shown in Figure 6.

4. CONCLUSIONS

A generalized computational procedure for predicting two phase flows in parallel, reverse and mixed flow manifold systems has been described. The capabilities of the computational procedure have been demonstrated by predicting an experimental situation describing the flow of an air-water mixture in a combining flow manifold. A reasonable agreement has been achieved. The flow rate distribution in branch pipes for different heat loads and different incoming flows has been evaluated theoretically for the parallel, reverse and mixed flow manifold system. It has been observed that the mixed flow manifold offers the maximum uniformity in lateral flow distribution. Although comparison of the computational procedure for parallel, reverse and mixed flow manifolds with experiment has not been possible, largely because of non-availability of published experimental data, the trend predicted by the model appears to be realistic. Further investigation should be directed to evaluation of the empirical constants C_T and C_d in two phase flows.

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REFERENCES

- 1. R. A. Bajura, A model for flow distribution in manifolds, *Tr ans. Am . Soc. M ech. Engrs.* Serie s *A,J. Engng Pwr93, 7-* 12 (1971).
- 2. A. Acrivos, B. D. Babcock and R. L. Pigford, Flow distribution in manifolds, *Chem. Engng Sci*. 10, 112-124 (1959).
- 3. T. Kubo and T. Ueda, On the characteristics of divided flow and confluent flow in headers, *Bull. JSME* 12, 802-809 (1969).
- 4. A. K. Majumdar, Mathematical modelling of flows in dividing and combining flow manifold, *Appl. Math. Model.* 4,424-432 (1980).
- 5. A. B. Datta and A. K. Majumdar, Flow distribution in parallel and reverse flow m an ifolds, *Int. J. Heat Fluid Flow* 2,253-262 (1980).
- 6. S. V. Patankar and D. B. Spalding, A calculation procedure for heat, mass and momentum transfer in three dimensional parabolic flows, *Int . J. lleat Mass Transfer* 15,1787-1806 (1972).
- 7. T. Kubo and T. Ueda, On the characteristics of confluent flowof ga s liquid mi xturesin headers,*Bull.J SME* 16, 1376- 1384 (1973).
- 8. J. G. Collier, *Conrectire Boiling and Condensation.* McGraw-Hill, New York (1972).
- 9. D. B. Spalding, Basic equations of fluid mechanics and heat and mass transfer, and procedures for their pollution, Imperial College, Mech. Eng. Dept., Report No. HTS/76/6 (1976).

APPENDIX

DERIVATION OF PRESSURE AND VELOCITY CORRECTION EQUATION

In order to make the velocity field compatible with continuity, there is a need to correct the pressure at each point. These pressure corrections $(p'_i$ and p'_{i+1}) lead to the correction of velocities, u'_1 which is given by

$$
u'_{i} = \frac{\partial u_{i}}{\partial p_{i}} p'_{i} + \frac{\partial u_{i}}{\partial p_{i+1}} p'_{i+1}.
$$

\n
$$
\frac{\partial u_{i}}{\partial p_{i}} \quad \text{and} \quad \frac{\partial u_{i}}{\partial p_{i+1}}
$$
 (A1)

are obtained from equation (19). Hence the velocity correction equation can be written as

$$
u'_{i} = \frac{a_{11}}{D_{i}^{u}} (p'_{i} - p'_{i+1}).
$$
 (A2)

The mass balance equation [Fig. 2(b)] can be written as

$$
\rho a_{\rm II}(u_i^* + u_i') - \rho a_{\rm II}(u_{i-1}^* + u_{i-1}') + q_i = 0. \tag{A3}
$$

Now the pressure correction equation [equation (25)] is obtained by substituting equation (A2) in equation $(A3)$.

UNE PROCEDURE DE CALCUL DE LA REPARTITION D'UN ECOULEMENT DIPHASIQUE DANS DES DISTRIBUTIONS AVEC OU SANS TRANSFERT THERMIQUE

Résumé-Une procédure de différence finie est développée pour calculer les écoulements diphasiques avec transfert thermique dans des distributeurs. Les ecoulements sont supposes homogenes et monodimensionnels. Un schema numerique iteratif resout les equations differentielles pour la continuite et la quantite de mouvement longitudinale tandis que, dans la direction laterale, on resout une equation integrale pour la quantite de mouvement. Les previsions numeriques se comparent bien avec les resultats expérimentaux disponibles pour un distributeur à écoulement combiné. L'effet du débit total et de l'entrée de chaleur sur les caractéristiques de distribution latérale et de perte de pression est démontré.

EINE BERECHNUNGSMETHODE FÜR DIE MASSENSTROMVERTEILUNG BEI ZWEIPHASENSTRÖMUNG IN VERTEILERROHREN MIT UND OHNE WÄRMEÜBERTRAGUNG

Zusammenfassung-Für die Berechnung einer Zweiphasenströmung mit Wärmeübergang in Rohrverteilern bei paralleler, umgekehrter und gemischter Strömung wurde ein finites Differenzenverfahren entwickelt. Es wird angenommen, daß die Strömung homogen und eindimensional ist. Die Differentialgleichungen für Impuls und Kontinuität in Längsrichtung werden iterativ numerisch gelöst, während in Querrichtung eine Integralgleichung fur den Impuls gelost wird. Die numerischen Ergebnisse stimmen gut mit den zur Verfiigung stehenden experimentellen Ergebnissen fur Sammler iiberein. Ferner werden der Einfiul3 des Gesamtmassenstroms, der Warmebelastung aufdie Stromungsverteilung in Querrichtung und das Verhalten des Druckabfalls dargestellt.

МЕТОД РАСЧЕТА РАСПРЕДЕЛЕНИЙ ДВУХФАЗНОГО ПОТОКА В ТРУБОПРОВОДАХ С УЧЕТОМ И БЕЗ УЧЕТА ТЕПЛОПЕРЕНОСА

Аннотация-Разработан конечно-разностный метод для расчета двухфазных потоков с теплообменом в трубопроводах при параллельных, возвратных и смешанных течениях. Предполагается, что потоки однородные и одномерные. Итерационная численная схема дает решение дифференциальных уравнений неразрывности и продольного импульса, тогда как поперечный импульс определяется из решения интегрального уравнения. Результаты численных расчетов хорошо согласуются с имеющимися экспериментальными данными для сложных течений в трубах. Показано влияние полного расхода и подвода тепла на поперечное распределение скорости и перепад давления.